Classical Athens counts

Serafina Cuomo

Serafina Cuomo is interested in what areas such as ancient account-keeping, mechanics, engineering, land surveying, and war engines can teach us about society and culture in the ancient world. Here she considers how ancient Athenians did their sums.

Numeracy has not received the same attention on the part of historians as literacy, even though several ancient sources seem to assume that in places like Classical Athens, the average inhabitant had at least some counting and calculating skills. Trade and even barter required counting and measuring (which in its turn implied counting); architecture, land division and distribution, and ordinary housekeeping may have required higher or lower levels of numeracy, but numeracy nonetheless. And Athenian citizens may also have needed a level of numeracy to play their full part in the democracy.

In a comic play that makes fun of the jury trial, a fundamental democratic institution, a young Athenian asks his father (who is obsessed with doing jury duty) carefully to consider the financial side of things:

> *Listen to me, dear little father, unruffle that frowning brow and calculate, you can do so without trouble, not with pebbles, but on your fingers, what is the sum-total of the tribute paid by the allied towns; besides this we have the direct taxes, a mass of percentage dues, the fees of the courts of justice, the produce from the mines, the markets, the harbours, the public lands and the confiscations. All these together amount to nearly two thousand talents. From this set aside the annual pay of the jurors; they number 6,000, and there have never been more in this town; so a total of 150 talents comes to us.*
>
> *Aristophanes,* Wasps *655–63 (after O'Neill's translation)*

If you do the sums, Aristophanes seems to suggest, you may get a better grasp of the actual state of things in Athens, than if you simply listened to rhetorically astute speeches.

Reconstructing ancient numeracy requires an unlearning of the way we do mathematics today, and our instinct to think of maths as a written domain: we have to imagine a situation with no institutionalized system of education, no standardized tests, no consensus about how mathematics ought, or ought not, to be done.

The average Athenian of the classical period probably conceived of numbers in quite concrete terms, as a quantity of objects or a sum of money, and never wrote calculations down. The high value that modern education attaches to mental arithmetic might have been baffling to him, or her. One scholar of ancient mathematics, Reviel Netz, has pointed out:

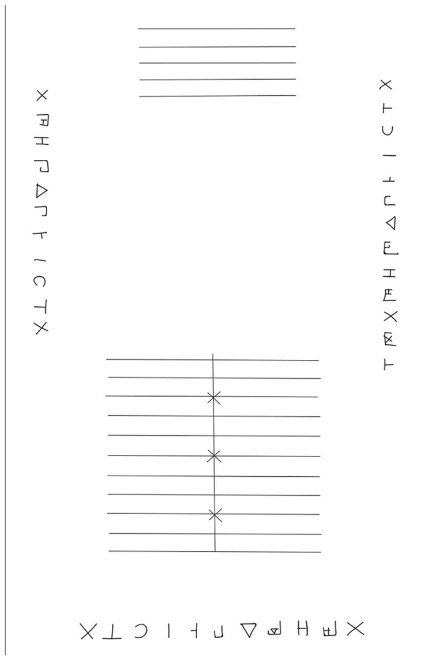
> *We imagine numbers as an entity seen on the page; the Greeks imagined them as an entity grasped between the thumb and the finger.*

We see something of this in the passage of Aristophanes, which mentions two methods of calculation: using fingers, and pebbles. Finger-counting may have used knuckles, the palm, and the wrist to indicate units, tens, hundreds, and so on. Calculation with pebbles required a flat surface with enough space to lay out the counters and move them around. Often the surface was an abacus, a table or board, which could be inscribed with numerals as well as parallel lines. Crucially, neither fingers nor pebbles involved writing; they were both, in different ways, manual, eminently physical sorts of calculation.

The abacus

Around thirty objects from the ancient world have been identified as counting boards, or abaci. We don't always know where they come from, but the vast majority we do know about came from the Greek-speaking part of the Mediterranean, with clusters in Attica, and towns like Oropus and Goritsa. All are made of marble or similar stone, and all are relatively large, in the shape either of slabs, or of self-standing tables.

The most famous example is the so-called Salamis table:



The Salamis table, now in the National Museum of Athens, has two sets of inscribed lines: a main one, with eleven parallel lines, cut by a perpendicular, with regularly spaced crosses at points of intersection, and a subsidiary set, with five smaller parallel lines. Three of the sides are inscribed with numerals (the ancient Greeks tended to use 'acrophonic' numerals, which means using the first letter of the word for a number – so Π for πέντε (five), Δ for δέκα (ten) and so on).

The Salamis abacus is an especially elaborate example. Other extant counting boards may have sets of lines, but no numerals, or a row of numerals, but no lines, or simply not be as well-preserved and detailed in their design. Some have a cavity, which may have been used to keep the pebbles or tokens used in the calculation.

Thus, the pebbles that Aristophanes' character is talking about would have been laid out in the columns of a board similar to the Salamis table.

Using an abacus

But how were the actual calculations carried out? Leaving aside finger counting for the time being, we can argue that pebble calculation on an abacus can be reconstructed with a degree of plausibility, on the basis of

1) brief mentions in some ancient texts
2) comparative evidence of abacus calculation from different times and places
3) archaeological remains.

Here, for example, is how an ancient philosophical writer makes a passing reference to the use of an abacus:

> *[Solon] used to say that those who had influence with tyrants were like the pebbles employed in calculations; for, as each of the pebbles represented now a large and now a small number, so the tyrants would treat each one of those about them at one time as great and famous, at another as of no account. (Diogenes Laertius,* Lives of the Philosophers *I.59, trans. Hicks)*

We see here that the numerical value of the pebble depended on its position on the abacus.

An abacus like that of Salamis had two sets of columns. Moving a pebble across columns changed its value, as Diogenes suggests. Like us, the Greeks counted in a base-ten system, so in the abacus the lower set of columns represented, right to left, units, tens, hundreds, and so on. The upper set immediately above represented 'quinaries' – that is, multiples of five, such as five, fifty, five hundred. Up to four pebbles could be placed in a lower column; adding one more would reach a 'quinary', and mean putting a pebble in the upper column instead. To signify six, a pebble could be placed in the units row to add to the 'five' in the quinary column above, and so on.

Addition and subtraction

In order to add two numbers, we think that the calculator would have taken pebbles for the first number and laid them out in the columns of the abacus. They then took pebbles for the second number, and also laid them out in the columns of the abacus. The next step would have been substitution. Every five pebbles in a ten-based, lower, column would be taken out and substituted by one pebble in the corresponding quinary column above. Equally, every two pebbles in a quinary column would be taken out and substituted by one pebble in the next 'units' column to the left (so two fives make 10, or two fifties make 100, and so on). When all the substitutions had been carried out, and there were no more than four pebbles in a lower column and no more than one pebble in an upper column, the calculator could simply read out the resulting number. That was the sum of the two initial numbers.

Subtractions might have necessitated additional substitution of one pebble from a higher column for five or two pebbles in a lower column, a procedure equivalent to the 'borrowing' or 'regrouping' taught in elementary schools today. Several possibilities must have existed for multiplications and divisions, from simple ones like repeated addition and repeated subtraction, respectively, to more complex methods which required knowledge of time tables, and/or of doubling and halving. This might explain the number of columns on the Salamis abacus. It is unlikely that its users would have wanted to count into the billions, so the 'spare' columns to the left may have been for use when the calculator needed to 'hold' some numbers, or to calculate intermediate steps towards the overall solution of a complex calculation.

In all these cases, making the calculation was a physical process, strengthening Netz's idea that the Greeks thought of numbers as tangible things.

Every vote counts: democracy and calculation

The passage of Aristophanes with which we opened suggested that the operations of the Athenian democracy, based as they were, at least nominally, on decisions taken by the majority, presupposed a numerate citizen body. Accounts, from those of the Parthenon to those of property sold at state auctions, to the tribute paid by Athens' allies, were publicly displayed in the form of inscriptions; citizens who had served the *polis* in an official capacity could be asked openly to render accounts at the end of their office.

The opportunities for the public – indeed, for the political – exercise of numeracy were everywhere. The symbolic power of scrutinizing accounts was not lost on the orator Aeschines:

> *Permit me to suggest how you ought to listen to the rest of my argument: as when we take our seats for the calculations of expenditures which extend back a long time, perhaps sometimes we set out from home having false opinions; but nevertheless, whenever the calculations are added up, nobody is so stubborn as to refuse, before leaving, to agree that whatever the calculation obtains, is the truth. I ask you to give a similar hearing now. (Aeschines,* Against Ctesiphon *59–60, trans. Adams.)*

The image of jurors as calculators must have been so effective that Aeschines' enemy Demosthenes returned to it in his speech, vehemently arguing the opposite case:

> *Then he [Aeschines] resorts to sophistry, and tells you that […], as, when you calculate a man's riches, if the counters are clean and nothing is left over, you assent, in this way now you must accept what is evident from the account. […] I shall prove without difficulty that he has no right to ask you to reverse that opinion – not by using counters, for the calculation of these matters is not like that, but by reminding you briefly of the several transactions, and appealing to you who hear me as both the witnesses and the accountants. (Demosthenes,* On the Crown *227, 229, trans. Vince and Vince.)*

Friends to count on

In both of these examples and in Aristophanes' earlier passage, calculation is portrayed as a collective, communal enterprise, even when it involves only a father and son, working out the income and expenditure of the Athenian *polis* together. The truth of such calculations is presented as indisputable – we are after all talking about mathematics – but strikingly, it is a truth which is arrived at by communication, rather than in isolation.

The shape and design themselves of the surviving abaci facilitated this shared nature of calculations: unlike their Roman counterparts (Roman abaci are the size of a human hand), Greek counting boards were large enough for the pebbles to be visible to more than one person. With its three strings of numerals facing outwards, the Salamis table invited people to witness the proceedings. It assumed more than one participant in the operation. In fact, some reconstructions of how multiplications and divisions were carried out allow for the presence of a human aid, a person who kept intermediate steps in the calculation on their fingers or in their heads, or, more simply, provided an extra pair of eyes, as in these vignettes of fourth-century Athenian life:

> *The absent-minded man is the sort who, when he has made a calculation with an abacus and determined the totals, asks the person sitting by him, 'What's the answer?' [...]*
>
> *When people [the fraudulent man] doesn't know are sitting beside him, he asks one of them to move the stones for him, and doing the addition from the thousands column to the ones and convincingly supplying names for each of these sums, he actually reaches ten talents. (Theophrastus,* Characters *14.1–2, 23.6, trans. Edmonds)*

In conclusion, Athenian counting and calculation were material rather than abstract, performative and oral rather than written, collective rather than individual. While calculation could be a matter for specialists, it was also arguably necessary for anyone who wanted to participate fully in the political life of the *polis*: counting really counted.

Serafina Cuomo is Professor of Ancient History at Durham.